An Introduction To Chaos

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What is Chaos Theory?

- ✍✍The study of non-linear systems;
  - ✍✍Weather Systems.
  - ✍✍Fluid Dynamics.
  - ✍✍Damped, Driven Systems.

- ✍✍The possibility of making long term predictions about a particular system.

- ✍✍Precision of measurement.
  - ✍✍No measurement can be made infinitely accurately.
  - ✍✍Will the results of a calculation increase in accuracy in proportion with the starting values?

- ✍✍Hallmarks of a chaotic system.
  - ✍✍Sensitivity to initial conditions.
  - ✍✍Bifurcations.

How and where does Chaos appear?

- ✍✍Chaotic properties can be found in even the simplest non-linear systems;
  - ✍✍*The Logistic Map*.

    $$x_{n+1} = ax_n(1-x_n) \qquad n=0,1,2,...$$

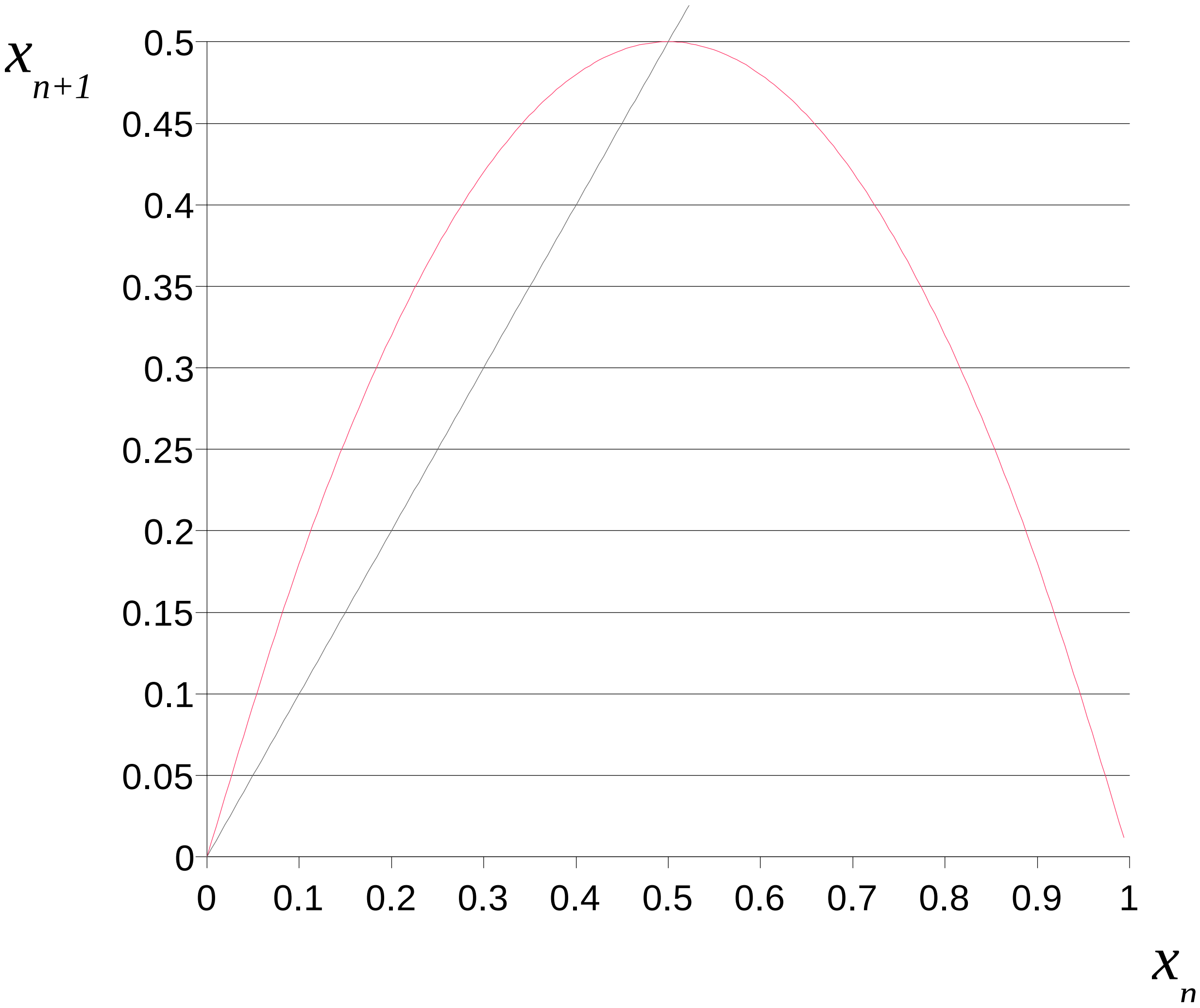
  - ✍✍*The Henon Map*.
- ✍✍Maps
  - ✍✍Mathematical functions.
  - ✍✍Inputs generate outputs which can, in turn, be used to generate more outputs?

The Logistic Map by way of example

- ✍✍ $x_{n+1} = ax_n(1-x_n)$ $n=0,1,2,...;$
  - ✍✍ Suppose that x_n describes the position of an object at time n.
  - ✍✍ *n* can take integer values only.
  - ✍✍ *a* is a parameter that describes the environment.
- ✍✍ Measure x_0 and *a* at $n=0$.
  - ✍✍ Precision of measurement.
- ✍✍ Suppose that x_n describes the position of an object at time n.
- ✍✍ Iterate the function to find out the position of the object after n time periods.
- ✍✍ Compare the results for different *initial values*.

Measurement of initial values

- $x_{n+1} = ax_n(1-x_n)$ $n=0,1,2,...;$
  - Let $a=2$.
  - Let $x_0=0.4$.



Iteration of the map

- $x_{n+1} = ax_n(1-x_n)$ $n=0,1,2,...;$
  - Let $a=2$.
  - Let $x_0=0.4$.

x_n	x_{n+1}
0.4	0.48
0.48	0.5
0.5	0.5
0.5	0.5
0.5	0.5
0.5	0.5
0.5	0.5
0.5	0.5
0.5	0.5
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0.5	0.5
0.5	0.5
0.5	0.5
0.5	0.5
0.5	0.5
0.5	0.5
0.5	0.5

Comparison with other values of *a*

✍✍ $x_{n+1} = ax_n(1-x_n)$ $n=0,1,2,\ldots;$

✍✍ Let $a=2$.
✍✍ Let $x_0=0.4$.

x_n	x_{n+1}
0.4	0.48
0.48	0.5
0.5	0.5
0.5	0.5
0.5	0.5
0.5	0.5
0.5	0.5
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0.5	0.5
0.5	0.5
0.5	0.5
0.5	0.5

✍✍ Let $a=3.5$.
✍✍ Let $x_0=0.4$.

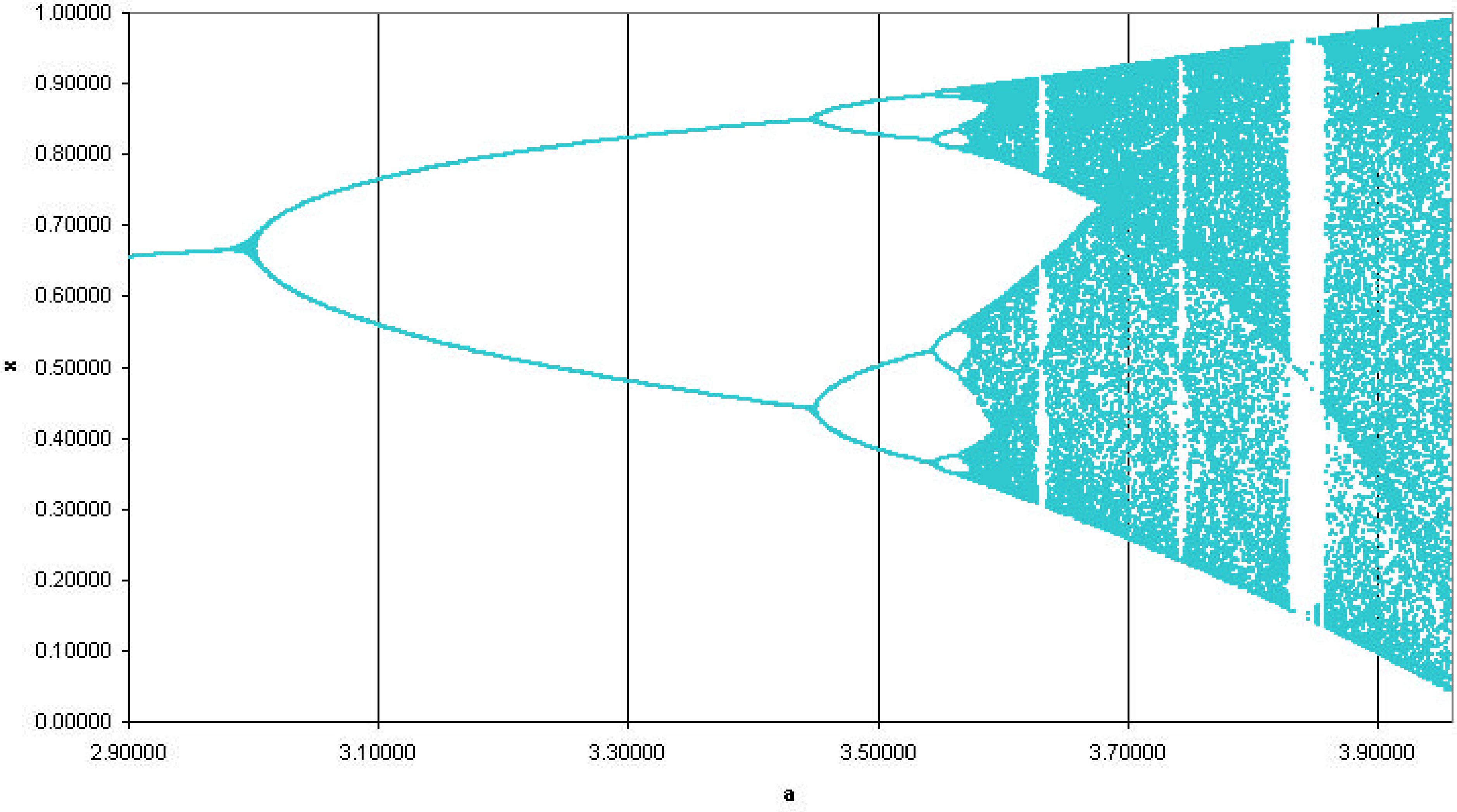
x_n	x_{n+1}
0.4	0.84
0.84	0.47
0.47	0.87
0.87	0.39
0.39	0.83
0.83	0.49
0.49	0.87
0.87	0.38
0.38	0.83
0.83	0.5
0.5	0.87
0.87	0.38
0.38	0.83
0.83	0.5
0.5	0.87
0.87	0.38
0.38	0.83
0.83	0.5

✍✍ Let $a=4$.
✍✍ Let $x_0=0.4$.

x_n	x_{n+1}
0.4	0.96
0.96	0.15
0.15	0.52
0.52	1
1	0.01
0.01	0.03
0.03	0.1
0.1	0.36
0.36	0.92
0.92	0.3
0.3	0.84
0.84	0.55
0.55	0.99
0.99	0.03
0.03	0.13
0.13	0.47
0.47	1
1	0.02

Attractors for a range of *a*

✍ ✍ $x_{n+1} = ax_n(1-x_n)$ $n=0,1,2,...;$



The Henon Map